4. Halla, por la definición, la derivada de $f(x) = \frac{3-x}{x-2}$ en el punto $x_0 = 1$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = (*)$$

$$f(1+h) = \frac{3-(1+h)}{1+h-2} = \frac{3-1-h}{-1+h} = \frac{2-h}{-1+h}$$

$$f(1) = \frac{3-1}{1-2} = \frac{2}{-1} = -2$$

$$(*) = \lim_{h \to 0} \frac{\frac{2-h}{-1+h} - (-2)}{h} = \lim_{h \to 0} \frac{\frac{2-h}{-1+h} + 2}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{2-h+2(-1+h)}{-1+h}}{h} = \lim_{h \to 0} \frac{\frac{\cancel{2}-h-\cancel{2}+2h}{-1+h}}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{(-1+h)\cancel{h}} = \lim_{h \to 0} \frac{1}{-1+h} = \frac{1}{-1} = \boxed{-1}$$

5. Calcula las siguientes derivadas

a) $f(x) = e^{2x} \log_3(5x^2+x)$

$$f'(x) = (e^{2x})' \cdot \log_3(5x^2+x) + \left(\log_3(5x^2+x)\right)' \cdot e^{2x} =$$

$$= e^{2x}(2x)' \cdot \log_3(5x^2+x) + \frac{(5x^2+x)'}{5x^2+x} \cdot \frac{1}{L3} \cdot e^{2x} =$$

$$= \boxed{e^{2x} \cdot 2 \cdot \log_3(5x^2+x) + \frac{10x+1}{5x^2+x} \cdot \frac{1}{L3} \cdot e^{2x}}$$

b) $f(x) = (5x^3 + 2x - 3^{2x})^3$

$$f'(x) = 3(5x^3+2x-3^{2x})^2 (5x^3+2x-3^{2x})' =$$

$$= \boxed{3(5x^3+2x-3^{2x})^2 \cdot (15x^2+2-2\cdot 3^{2x} \cdot L3)}$$

c) $f(x) = L^4(3x^3+2x^2)$

$$f'(x) = 4 \cdot L^3(3x^3+2x^2) \cdot \left(L(3x^3+2x^2)\right)' =$$

$$= 4 \cdot L^3(3x^3+2x^2) \cdot \frac{(3x^3+2x^2)'}{3x^3+2x^2} =$$

$$= \boxed{4 \cdot L^3(3x^3+2x^2) \cdot \frac{9x^2+4x}{3x^3+2x^2}}$$

d) $f(x) = \dfrac{4x^2-1}{L(x+2)}$

$$f'(x) = \frac{(4x^2-1)' \cdot L(x+2) - \left(L(x+2)\right)' \cdot (4x^2-1)}{L^2(x+2)} =$$

$$= \frac{8x \cdot L(x+2) - \frac{(x+2)'}{x+2} \cdot (4x^2-1)}{L^2(x+2)} =$$

$$= \boxed{\frac{8x \cdot L(x+2) - \frac{1}{x+2} \cdot (4x^2-1)}{L^2(x+2)}}$$

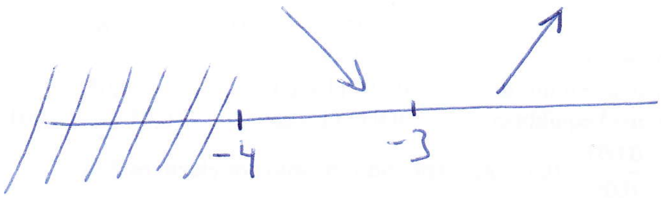
⑥

$f(x) = x+3 - L(x+4)$

$\mathrm{Dom}\, f(x) = \{x \in \mathbb{R} / x+4 > 0\} = (-4, \infty)$

$$f'(x) = 1 - \frac{(x+4)'}{x+4} = 1 - \frac{1}{x+4} = \frac{x+4-1}{x+4} = \frac{x+3}{x+4}$$

Signo $\dfrac{x+3}{x+4}$ $\begin{cases} \text{ceros} & x+3=0\ ;\ x=-3 \\ \text{polos} & x+4=0\ ;\ x=-4 \end{cases}$



$f'(-3'5) = \dfrac{-3'5+3}{-3'5+4} = \dfrac{-0'5}{0'5} = -1 < 0 \Rightarrow$ decreciente.

$f'(0) = \dfrac{0+3}{0+4} = \dfrac{3}{4} > 0 \Rightarrow$ creciente

$(-4,-3)$ decreciente
$(-3,\infty)$ creciente
mínimo relativo en $(-3, f(-3))$

⑦ $f(x) = x^3 - 3x^2 + 2$

$R_T : \begin{cases} P(2, f(2)) \rightarrow (2,-2) \\ m_{RT} = f'(2) \end{cases}$

$f'(x) = 3x^2 - 6x$

$f'(2) = 3\cdot 4 - 6\cdot 2 = 0 \Rightarrow m_{RT} = 0$

$R_T :\ y - (-2) = 0(x-2)$

$\boxed{y + 2 = 0}$