① Resuelto en página 42 del libro.

② $P(x) = 2x^3 - 2x^2 - 2x - 4$
$Q(x) = -2x^2 - x + 10$

$$\begin{array}{c|cccc} & 2 & -2 & -2 & -4 \\ 2 & & 4 & 4 & 4 \\ \hline & 2 & 2 & 2 & \boxed{10} \end{array}$$

$P(x) = (x-2)(2x^2+2x+2) = 2(x-2)(x^2+x+1)$

$2x^2+2x+2=0; \quad x = \frac{-2 \pm \sqrt{4-4\cdot 2\cdot 2}}{4} = \frac{-2 \pm \sqrt{-12}}{4} \notin \mathbb{R}$

$-2x^2 - x + 10 = 0$

$x = \frac{1 \pm \sqrt{1+80}}{-4} = \frac{1 \pm 9}{-4} = \begin{cases} 2 \\ -\frac{5}{2} \end{cases}$

$Q(x) = -2(x-2)\left(x+\frac{5}{2}\right)$

$$\boxed{\begin{array}{l} MCD(P(x), Q(x)) = 2(x-2) \\ m.c.m(P(x), Q(x)) = 2(x-2)\left(x+\frac{5}{2}\right)(x^2+x+1) \end{array}}$$

③ $\left(3x - \frac{1}{x}\right)^4 = \binom{4}{0}(3x)^4 + \binom{4}{1}(3x)^3\left(-\frac{1}{x}\right) + \binom{4}{2}(3x)^2\left(-\frac{1}{x}\right)^2 + \binom{4}{3}3x\left(-\frac{1}{x}\right)^3 + \binom{4}{0}\left(-\frac{1}{x}\right)^4 =$

$= 81x^4 + 4\cdot 27x^3\cdot\left(-\frac{1}{x}\right) + 6\cdot 9x^2\frac{1}{x^2} + 4\cdot 3\cdot\left(-\frac{1}{x^3}\right) + \frac{1}{x^4} =$

$= \boxed{81x^4 - 108x^2 + 54 - \frac{12}{x^3} + \frac{1}{x^4}}$

④ a) $\sqrt{x+3} + \sqrt{x+6} = \frac{3}{\sqrt{x+3}}$

$\left(\sqrt{x+3} + \sqrt{x+6}\right)\sqrt{x+3} = 3; \quad \left(\sqrt{x+3}\right)^2 + \sqrt{x+6}\sqrt{x+3} = 3$

$x + \cancel{3} + \sqrt{(x+6)(x+3)} = \cancel{3}; \quad \left(\sqrt{x^2+9x+18}\right)^2 = (-x)^2; \quad x^2+9x+18 = x^2$

$9x + 18 = 0; \quad \boxed{x = -2}$

b) $\left.\begin{array}{l} \frac{2}{x} + \frac{3}{y} = 7 \\ \frac{3}{x} + \frac{2}{y} = \frac{11}{2} \end{array}\right\} \quad \begin{array}{r} \frac{6}{x} + \frac{9}{y} = 21 \\ -\frac{6}{x} - \frac{4}{y} = -11 \\ \hline \end{array}$

$\frac{5}{y} = 10; \quad 5 = 10y; \quad y = \frac{5}{10} = \frac{1}{2}; \quad \boxed{y = \frac{1}{2}}$

$\frac{2}{x} + \frac{3}{1/2} = 7; \quad \frac{2}{x} + 6 = 7; \quad \frac{2}{x} = 1; \quad \boxed{x = 2}$

c) $5^{x+1} = 10 + 3 \cdot 5^{2-x}$

$5 \cdot 5^x = 10 + 3 \cdot \frac{5^2}{5^x}$; $5 \cdot 5^x = 10 + \frac{75}{5^x}$; $5^x = z$

$5z = 10 + \frac{75}{z}$; $5z^2 = 10z + 75$; $5z^2 - 10z - 75 = 0$; $z^2 - 2z - 15 = 0$

$z = \frac{2 \pm \sqrt{4+60}}{2} = \frac{2 \pm \sqrt{64}}{2} = \frac{2 \pm 8}{2} = \begin{cases} 5 \\ -3 \end{cases}$

$5^x = z \begin{cases} 5^x = 5 \; ; \; \boxed{x = 1} \\ 5^x \neq -3 \end{cases}$

d) $\left.\begin{matrix} e^x = \frac{e^{11}}{e^y} \\ \log(x+y) + \log(x-y) = \log 55 \end{matrix}\right\}$ $\left.\begin{matrix} e^x = e^{11-y} \\ (x+y)(x-y) = 55 \end{matrix}\right\}$ $\left.\begin{matrix} x = 11 - y \\ x^2 - y^2 = 55 \end{matrix}\right\}$

$(11-y)^2 - y^2 = 55$; $121 + \cancel{y^2} - 22y - \cancel{y^2} = 55$

$-22y = -66$; $\boxed{y = 3} \Rightarrow \boxed{x = 8}$

e) $\cancel{5^{x+1} = 10 + 3 \cdot 5^{2-x}}$

$\cos 2x - \cos 6x = \operatorname{sen} 5x + \operatorname{sen} 3x$

$-2 \operatorname{sen} 4x \cdot \operatorname{sen}(-2x) = 2 \operatorname{sen} 4x \cos x$

$-\cancel{2} \operatorname{sen} 4x \cdot (-\operatorname{sen} 2x) = \cancel{2} \operatorname{sen} 4x \cos x$

$\operatorname{sen} 4x \operatorname{sen} 2x = \operatorname{sen} 4x \cos x$; $\operatorname{sen} 4x \operatorname{sen} 2x - \operatorname{sen} 4x \cos x = 0$

$\operatorname{sen} 4x (\operatorname{sen} 2x - \cos x) = 0 \begin{cases} \operatorname{sen} 4x = 0 \\ \operatorname{sen} 2x - \cos x = 0 \end{cases}$

$\operatorname{sen} 4x = 0$; $4x = \operatorname{arcsen} 0$; $4x = \begin{cases} 0 + 360K \\ 180 + 360K \end{cases}$;

$4x = 0 + 180K$; $\boxed{x = 0 + 45K}$

$\operatorname{sen} 2x - \cos x = 0$; $2 \operatorname{sen} x \cos x - \cos x = 0$; $\cos x (2 \operatorname{sen} x - 1) = 0 \begin{cases} \cos x = 0 \\ 2 \operatorname{sen} x - 1 = 0 \end{cases}$

$\cos x = 0$; $x = \arccos 0$; $x = \begin{cases} 90 + 360K \\ 270 + 360K \end{cases}$; $\boxed{x = 90 + 180K}$

$2 \operatorname{sen} x - 1 = 0$; $\operatorname{sen} x = \frac{1}{2}$; $x = \operatorname{arcsen} \frac{1}{2} = \begin{cases} 30 + 360K \\ 150 + 360K \end{cases}$; $\boxed{x = \begin{cases} 30 + 360K \\ 150 + 360K \end{cases}}$

f) $6\cos^2\frac{x}{2}+\cos x=1$; $6\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2+\cos x=1$;

$6\,\frac{1+\cos x}{2}+\cos x=1$; $3(1+\cos x)+\cos x=1$; $3+3\cos x+\cos x=1$

$4\cos x=-2$; $\cos x=-\frac{1}{2}$; $\boxed{x=\arccos -\frac{1}{2}=\begin{cases}120+360k\\240+360k\end{cases}}$

(5) $a=30m$ $\hat{A}=40°$
$b=40m$

$\frac{a}{\operatorname{sen}\hat{A}}=\frac{b}{\operatorname{sen}\hat{B}}$; $\frac{30}{\operatorname{sen}40}=\frac{40}{\operatorname{sen}\hat{B}}$; $\operatorname{sen}\hat{B}=\frac{40\cdot\operatorname{sen}40}{30}$

$\hat{B}=\operatorname{arc\,sen}\frac{40\cdot\operatorname{sen}40}{30}=\begin{cases}59°\to\hat{B}_1=59°\\121°\to\hat{B}_2=121°\end{cases}$

$\hat{C}_1=180-\hat{B}_1-\hat{A}=180-59-40=81°$

$\hat{C}_2=180-\hat{B}_2-\hat{A}=180-121-40=19°$

$\frac{a}{\operatorname{sen}\hat{A}}=\frac{c_1}{\operatorname{sen}\hat{C}_1}$; $\frac{30}{\operatorname{sen}40}=\frac{c_1}{\operatorname{sen}81}$; $c_1=46'1\,m$

$\frac{a}{\operatorname{sen}\hat{A}}=\frac{c_2}{\operatorname{sen}\hat{C}_2}$; $\frac{30}{\operatorname{sen}40}=\frac{c_2}{\operatorname{sen}19}$; $c_2=15'19\,m$

TRIÁNGULO 1:	$a=30$, $b=40$, $c=46'1$, $\hat{A}=40$, $\hat{B}=59$, $\hat{C}=81°$
TRIÁNGULO 2:	$a=30$, $b=40$, $c=15'19$, $\hat{A}=40$, $\hat{B}=121$, $\hat{C}=19°$

(6) $\operatorname{tg}2a=\sqrt{x}$; ¿tg a?

$\operatorname{tg}2a=\frac{2\operatorname{tg}a}{1-\operatorname{tg}^2a}=\sqrt{x}$; $2\operatorname{tg}a=\sqrt{x}-\sqrt{x}\operatorname{tg}^2a$;

$\sqrt{x}\operatorname{tg}^2a+2\operatorname{tg}a-\sqrt{x}=0$; $\operatorname{tg}a=\frac{-2\pm\sqrt{4+4\sqrt{x}\sqrt{x}}}{2\sqrt{x}}=\frac{-2\pm\sqrt{4+4x}}{2\sqrt{x}}$

$\operatorname{tg}a=\frac{-2\pm\sqrt{4(1+x)}}{2\sqrt{x}}$, $\operatorname{tg}a=\frac{-2\pm2\sqrt{1+x}}{2\sqrt{x}}$; $\boxed{\operatorname{tg}a=\frac{-1\pm\sqrt{1+x}}{\sqrt{x}}}$

7) $$\underbrace{\frac{\operatorname{sen} 3x + \operatorname{sen} x}{\operatorname{sen} 3x - \operatorname{sen} x}}^{\mathrm{I}} = \overbrace{\frac{2}{1+\operatorname{tg}^2 x}}^{\mathrm{II}}$$

$$\mathrm{I} = \frac{\cancel{2} \operatorname{sen} 2x \cos x}{\cancel{2} \cos 2x \operatorname{sen} x} = \operatorname{tg} 2x \operatorname{cotg} x = \frac{2\cancel{\operatorname{tg} x}}{1-\operatorname{tg}^2 x} \cdot \frac{1}{\cancel{\operatorname{tg} x}} = \frac{2}{1-\operatorname{tg}^2 x} \neq \mathrm{II}$$

NO SE VERIFICA LA IGUALDAD.

8

$r: 3x + 4y - 1 = 0$

$$s: \begin{cases} s \parallel r \\ d(r,s) = 3 \end{cases}$$

$\vec{v}_r = (-4,3) \Rightarrow \vec{v}_s = (-4,3)$

$s: 3x + 4y + C = 0$

Para calcular C, utilizo que $d(r,s) = 3$

$d(r,s) = d(A,s)$ siendo A un punto de r.

Hallo A.

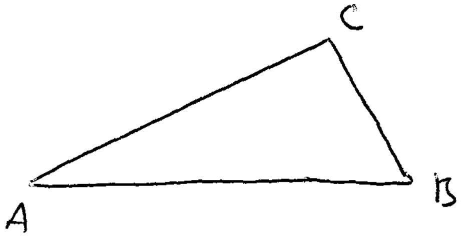
$y = 1 \Rightarrow 3x + 4 - 1 = 0;\ 3x + 3 = 0;\ x = -1;\ A(-1,1)$

$$d(r,s) = d(A,s) = \frac{|3(-1) + 4\cdot 1 + C|}{\sqrt{3^2+4^2}} = 3$$

$$\frac{|-3+4+C|}{\sqrt{25}} = 3; \quad |1+C| = 3\cdot 5$$

$$|1+C| = 15 \Rightarrow \begin{cases} \boxed{C = 14} \\ \boxed{C = -16} \end{cases}$$

9) $A(0,0)$
$B(4,-2)$
$C(-2,6)$



a) Área??

$$\text{Área} = \frac{\text{base} \cdot \text{altura}}{2} = \frac{|\vec{AB}| \cdot d(C,r)}{2}$$

siendo r la recta que contiene al lado AB.

$\vec{AB} = (4,-2)$

$|\vec{AB}| = \sqrt{16+4} = \sqrt{20}$

$$r \begin{cases} \vec{v}_r = \vec{AB} = (4,-2) \\ A(0,0) \end{cases} \qquad r: -2x - 4y + C = 0$$

$-2 \cdot 0 - 4 \cdot 0 + C = 0 \ ; \ C = 0$

$r: -2x - 4y = 0$

$$d(C,r) = \frac{|-2(-2) + (-4)\cdot 6|}{\sqrt{4+16}} = \frac{|4-24|}{\sqrt{20}} = \frac{|-20|}{\sqrt{20}} = \frac{20}{\sqrt{20}}$$

$$\text{Área} = \frac{|\vec{AB}| \cdot d(C,r)}{2} = \frac{\sqrt{20} \cdot \frac{20}{\sqrt{20}}}{2} = \frac{20}{2} = \boxed{10\ u^2}$$

b)

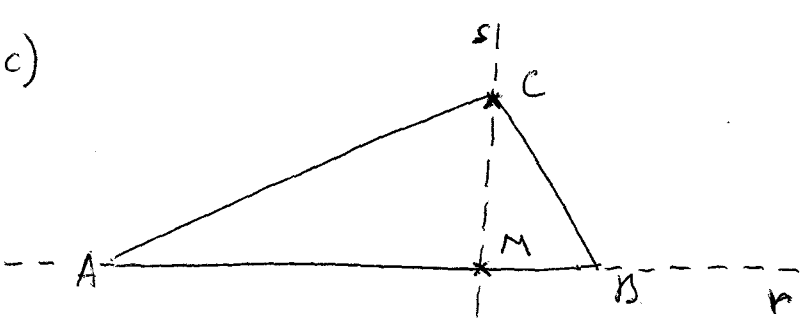
$$\hat{B} = \arccos \frac{\vec{BC} \cdot \vec{BA}}{|\vec{BC}|\,|\vec{BA}|}$$

$\vec{BC} = (-6,8) \qquad |\vec{BC}| = \sqrt{36+64} = \sqrt{100} = 10$

$\vec{BA} = (-4,2) \qquad |\vec{BA}| = \sqrt{16+4} = \sqrt{20}$

$$\hat{B} = \arccos \frac{-6(-4)+2\cdot 8}{10 \cdot \sqrt{20}} = \arccos \frac{40}{10\sqrt{20}} = \boxed{26'56^\circ}$$

c)



Hallo la recta s que es perpendicular a r y pasa por C.

$\vec{v}_r = \vec{AB} = (4,-2)$; $m_r = -\frac{1}{2}$

$$s: \begin{cases} C(-2,6) \\ s \perp r \Rightarrow m_s = -\frac{1}{m_r} ; \ m_s = 2 \end{cases}$$

$y - 6 = 2(x+2)$

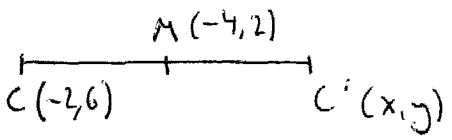
$y - 6 = 2x + 4$; $s: 2x - y + 10 = 0$

Hallo el punto M que es el de corte entre las rectas r y s.

$$\begin{array}{rl} r: & -2x - 4y = 0 \\ s: & 2x - y + 10 = 0 \\ \hline & -5y + 10 = 0 \end{array}$$

$-5y + 10 = 0$; $y = 2 \Rightarrow x = -4$ $\quad M(-4,2)$

Hallo el punto simétrico C'



$-4 = \frac{-2+x}{2} \Rightarrow x = -6$

$2 = \frac{6+y}{2} \Rightarrow y = -2$

$$\boxed{C' = (-6,-2)}$$

(10) $r: 5x - y + 4 = 0$

$$s: \left.\begin{array}{l} x = -3 + m\alpha \\ y = 4 - \alpha \end{array}\right\} \begin{array}{l} \alpha = \frac{x+3}{m} \\ \alpha = 4 - y \end{array}$$

$\frac{x+3}{m} = 4 - y$; $x + 3 = 4m - my$

$s: x + my + 3 - 4m = 0$

a) Paralelas $\frac{A}{A'} = \frac{B}{B'} \neq \frac{C}{C'}$

$\frac{5}{1} = \frac{-1}{m}$; $5m = 1$; $\boxed{m = \frac{1}{5}}$; Compruebo si son paralelas o

coincidentes: $\frac{A}{A'} = \frac{C}{C'}$; $\frac{5}{1} = \frac{4}{3 - 4\cdot\frac{1}{5}}$; $\frac{5}{1} = \frac{4}{\frac{15-4}{5}}$; $\frac{5}{1} = \frac{20}{16}$; $5 \cdot 16 \neq 20$

$\Downarrow$

PARALELAS

b) Secantes $\boxed{m \neq \frac{1}{5}}$