1) a) $2^x - 5\cdot 2^{-x} + 4\cdot 2^{-3x} = 0$; $2^x - \frac{5}{2^x} + \frac{4}{2^{3x}} = 0$; $2^x = z$; $z - \frac{5}{z} + \frac{4}{z^3} = 0$; $z^4 - 5z^2 + 4 = 0$

$z^2 = y$; $y^2 - 5y + 4 = 0$, $y = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$z^2 = y \begin{cases} z^2 = 4 ; & z = \pm 2 \\ z^2 = 1 ; & z = \pm 1 \end{cases}$ $\quad 2^x = z \begin{cases} 2^x = 2 \Rightarrow \boxed{x = 1} \\ 2^x \neq -2 \\ 2^x = 1 \Rightarrow \boxed{x = 0} \\ 2^x \neq -1 \end{cases}$

b) $\text{sen}\, 6x + \text{sen}\, 2x = \cos 3x - \cos 5x$

$2\,\text{sen}\, 4x \cdot \cos 2x = -2\,\text{sen}\, 4x\, \text{sen}(-x)$; $2\,\text{sen}\, 4x \cos 2x = 2\,\text{sen}\, 4x\, \text{sen}\, x$

$2\,\text{sen}\, 4x \cos 2x - 2\,\text{sen}\, 4x\, \text{sen}\, x = 0$; $2\,\text{sen}\, 4x\, (\cos 2x - \text{sen}\, x) = 0$

* $2\,\text{sen}\, 4x = 0$; $\text{sen}\, 4x = 0$; $4x = \text{arc sen}\, 0 = \begin{cases} 0 + 360k \\ 180 + 360k \end{cases} = 0 + 180k$

$x = \frac{0 + 180k}{4}$; $\boxed{x = 0^\circ + 45^\circ k}$

* $\cos 2x - \text{sen}\, x = 0$; $\cos^2 x - \text{sen}^2 x - \text{sen}\, x = 0$; $1 - \text{sen}^2 x - \text{sen}^2 x - \text{sen}\, x = 0$

$-2\,\text{sen}^2 x - \text{sen}\, x + 1 = 0$; $\text{sen}\, x = \frac{1 \pm \sqrt{1+8}}{-4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$

$x = \text{arc sen}\, -1 = \boxed{270 + 360k}$; $x = \text{arc sen}\, \frac{1}{2} = \begin{cases} \boxed{30 + 360k} \\ \boxed{150 + 360k} \end{cases}$

c) $6\cos^2 \frac{x}{2} + \cos x = 1$; $6\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 + \cos x = 1$

$6\,\frac{1+\cos x}{2} + \cos x = 1$; $3(1 + \cos x) + \cos x = 1$; $3 + 3\cos x + \cos x = 1$; $4\cos x = -2$

$\cos x = -\frac{1}{2}$; $x = \text{arc cos}\, -\frac{1}{2} = \begin{cases} \boxed{120 + 360k} \\ \boxed{240 + 360k} \end{cases}$

d) $\log(2x-1) - 3\log 2 = 2 + \log(x+2)$

$\log(2x-1) - \log 2^3 = \log 100 + \log(x+2)$,

$\log \frac{2x-1}{8} = \log[100(x+2)]$; $\frac{2x-1}{8} = 100x + 200$; $2x - 1 = 800x + 1600$

$2x - 800x = 1601$, $-798x = 1601$; $\boxed{x = -\frac{1601}{798}}$ NO VALE

e) $\left.\begin{array}{l} x^2 + y^2 = 25 \\ \log_2 2^x - \log_2 2^y = 1 \end{array}\right\}$ $\left.\begin{array}{l} x^2 + y^2 = 25 \\ x\log_2 2 - y\log_2 2 = 1 \end{array}\right\}$ $\left.\begin{array}{l} x^2 + y^2 = 25 \\ x - y = 1 \end{array}\right\} \rightarrow x = 1 + y$

$(1+y)^2 + y^2 = 25$; $1 + y^2 + 2y + y^2 = 25$; $2y^2 + 2y - 24 = 0$; $y^2 + y - 12 = 0$

$y = \frac{-1 \pm \sqrt{1+48}}{2} = \frac{-1 \pm 7}{2} = \begin{cases} 3 \\ -4 \end{cases}$ $\quad \boxed{\begin{array}{l} y = 3 \Rightarrow x = 4 \\ y = -4 \Rightarrow x = -3 \end{array}}$

(2) $$\frac{2x^3+x^2-2x-2}{x^2+x} = 2x-1+\frac{-x-2}{x^2+x}$$

$$\begin{array}{r|l} 2x^3+x^2-2x-2 & x^2+x \\ \cline{2-2} -2x^3-2x^2 & 2x-1 \\ \cline{1-1} -x^2-2x-2 & \\ x^2+x & \\ \cline{1-1} -x-2 & \end{array}$$

$$\frac{-x-2}{x^2+x} = \frac{-x-2}{x(x+1)} = \frac{A}{x}+\frac{B}{x+1} = \frac{(x+1)A+xB}{x(x+1)}$$

$-x-2 = (x+1)A + Bx$

si $x=0 \Rightarrow -2 = A$

si $x=-1 \Rightarrow -1 = -B$; $B=1$

$$\boxed{\frac{2x^3+x^2-2x-2}{x^2+x} = 2x-1+\frac{-2}{x}+\frac{1}{x+1}}$$

(3) $$(3x-x^2)^4 = \binom{4}{0}(3x)^4-\binom{4}{1}(3x)^3x^2+\binom{4}{2}(3x)^2(x^2)^2-\binom{4}{3}3x(x^2)^3+\binom{4}{4}(x^2)^4 =$$

$$= 1\cdot 81\cdot x^4 - 4\cdot 27x^3x^2 + 6\cdot 9x^2x^4 - 4\cdot 3x\,x^6 + 1\cdot x^8 =$$

$$= 81x^4 - 108x^5 + 54x^6 - 12x^7 + x^8$$

(4) $\hat{A}=40$; $a=30$ m; $b=40$ m

$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen}\hat{B}}$; $\frac{30}{\operatorname{sen} 40} = \frac{40}{\operatorname{sen}\hat{B}}$; $\operatorname{sen}\hat{B} = \frac{40\cdot\operatorname{sen}40}{30} = 0'86$; $\hat{B} = \begin{cases} 58'99° = \hat{B}_1 \\ 121'01° = \hat{B}_2 \end{cases}$

$\hat{C}_1 = 180 - 58'99 - 40 = 81'01°$

$\hat{C}_2 = 180 - 121'01 - 40 = 18'99°$

$\frac{A}{\operatorname{sen}\hat{A}} = \frac{C_1}{\operatorname{sen}\hat{C}_1}$; $\frac{30}{\operatorname{sen}40} = \frac{c_1}{\operatorname{sen}81'01}$; $c_1 = \frac{30\cdot\operatorname{sen}81'01}{\operatorname{sen}40} = 46'1$ m

$\frac{A}{\operatorname{sen}\hat{A}} = \frac{c_2}{\operatorname{sen}\hat{C}_2}$; $\frac{30}{\operatorname{sen}40} = \frac{c_2}{\operatorname{sen}18'99}$; $c_2 = \frac{30\cdot\operatorname{sen}18'99}{\operatorname{sen}40} = 14'8$ m

Triángulo 1:	$\hat{A}=40$; $\hat{B}_1 = 58'99°$; $\hat{C}_1 = 81'01°$; $a=30$; $b=40$; $c_1 = 46'1$
Triángulo 2	$\hat{A}=40$; $\hat{B}_2 = 121'01°$; $\hat{C}_2 = 18'99°$; $a=30$; $b=40$; $c_2 = 14'8$

(5) $\operatorname{tg} 2a = \sqrt{3}$; $\frac{2\operatorname{tg}a}{1-\operatorname{tg}^2a} = \sqrt{3}$; $2\operatorname{tg}a = \sqrt{3}(1-\operatorname{tg}^2a)$;

$2\operatorname{tg}a = \sqrt{3} - \sqrt{3}\operatorname{tg}^2a$; $\sqrt{3}\operatorname{tg}^2a + 2\operatorname{tg}a - \sqrt{3} = 0$; $\operatorname{tg}a = \frac{-2\pm\sqrt{4+4\cdot\sqrt{3}\cdot\sqrt{3}}}{2\sqrt{3}}$;

$$\operatorname{tg}a = \frac{-2\pm\sqrt{4+4\cdot 3}}{2\sqrt{3}} = \frac{-2\pm\sqrt{16}}{2\sqrt{3}} = \frac{-2\pm 4}{2\sqrt{3}} = \begin{cases} \frac{+2}{2\sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}} \\ \frac{-6}{2\sqrt{3}} = \boxed{\frac{-3}{\sqrt{3}}} \end{cases}$$

⑥ a) $\underbrace{tg\ x}_{I} = \underbrace{cotg\ x - 2\cot g\ 2x}_{II}$

$$II = \frac{1}{tg\,x} - \frac{2}{tg\,2x} = \frac{1}{tg\,x} - \frac{2}{\frac{2\,tg\,x}{1-tg^2x}} = \frac{1}{tg\,x} - \frac{\cancel{2}(1-tg^2x)}{\cancel{2}\,tg\,x} =$$

$$= \frac{1}{tg\,x} - \frac{1-tg^2x}{tg\,x} = \frac{1-1+tg^2x}{tg\,x} = \frac{tg^{\cancel{2}}x}{\cancel{tg\,x}} = tg\,x = I$$

b) $\underbrace{\dfrac{\cos(A-B)-\cos C}{2\cos A}}_{I} = \underbrace{\cos B}_{II}$ $\qquad A+B+C=180^\circ \Rightarrow C = 180-(A+B)$

$$I = \frac{\cos(A-B)-\cos(180-(A+B))}{2\cos A} = \frac{\cos A\cos B + \operatorname{sen}A\operatorname{sen}B - (\cos 180\cos(A+B) + \cancel{\operatorname{sen}180}^{\,0}\operatorname{sen}(A+B))}{2\cos A}$$

$$= \frac{\cos A\cos B + \operatorname{sen}A\operatorname{sen}B - (-1)\cos(A+B)}{2\cos A} =$$

$$= \frac{\cos A\cos B + \cancel{\operatorname{sen}A\operatorname{sen}B} + \cos A\cos B - \cancel{\operatorname{sen}A\operatorname{sen}B}}{2\cos A} = \frac{\cancel{2}\,\cancel{\cos}A\cos B}{\cancel{2}\,\cancel{\cos}A} = \cos B = II$$

⑦ $r:\ ax-2y+7=0$

$s:\ \dfrac{x+1}{b} = \dfrac{y}{2}$; $2x+2=by$; $2x-by+2=0$; $\quad s:\ 2x-by+2=0$

r pasa por $(-1,2) \Rightarrow a(-1)-2\cdot 2+7=0$; $-a-4+7=0$; $-a+3=0$; $\boxed{a=3}$

a) $\underbrace{\dfrac{a}{2} = \dfrac{-2}{-b}}$ $\neq \dfrac{7}{2}$

$\hookrightarrow -ab=-4$; $-3b=-4$; $\boxed{b=\frac{4}{3}}$

b) $\left.\begin{array}{l}\vec{v}_r = (2,a) = (2,3)\\ \vec{v}_s = (b,2)\end{array}\right\}$ si $r \perp s \Rightarrow \vec{v}_r\cdot\vec{v}_s = 0$; $2b+6=0$; $\boxed{b=-3}$

⑧ $4x+3y-12=0 \Rightarrow \vec{v} = (-3,4) \Rightarrow \vec{v}_\perp(4,3)$

La recta perpendicular a la dada será $r: 3x-4y+C=0$

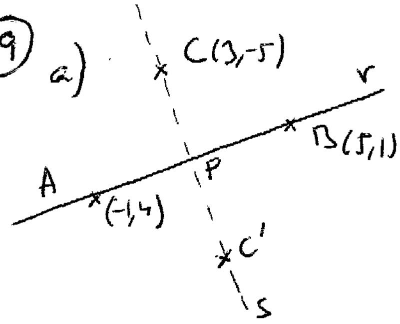
Hallo C sabiendo que la recta dista 5 unidades del punto $P(1,1)$

$$d(P,r) = \frac{|3\cdot 1 - 4\cdot 1 + C|}{\sqrt{3^2+4^2}} = 5\ ;\quad \frac{|3-4+C|}{\sqrt{25}} = 5\ ;\quad \frac{|-1+C|}{5} = 5\ ;\quad |-1+C| = 25$$

$-1+C=25$; $C=26 \Rightarrow$ $\boxed{r:\ 3x-4y+26=0}$

$-1+C=-25$; $C=-24 \Rightarrow$ $\boxed{r:\ 3x-4y-24=0}$

(9) a)



Hallo r:

$r\begin{cases} A(-1,4) \\ \vec{v}_r = \vec{AB} = B-A = (6,-3) \end{cases}$ $\quad \dfrac{x+1}{6} = \dfrac{y-4}{-3}$; $-3x-3 = 6y-24$

$r: -3x-6y+21=0$

Hallo s:

$s\begin{cases} C(3,-5) \\ \vec{v}_s = (3,6) \end{cases}$ $\quad \dfrac{x-3}{3} = \dfrac{y+5}{6}$; $6x-18 = 3y+15$

$s: 6x-3y-33=0$

Hallo el punto P que es el punto de corte de r y s.

$\left.\begin{array}{l} -3x-6y+21=0 \\ 6x-3y-33=0 \end{array}\right\}$ $\begin{array}{l} -6x-12y+42=0 \\ \underline{6x-3y-33=0} \\ -15y+9=0 \end{array}$; $y = \dfrac{9}{15} = \dfrac{3}{5}$

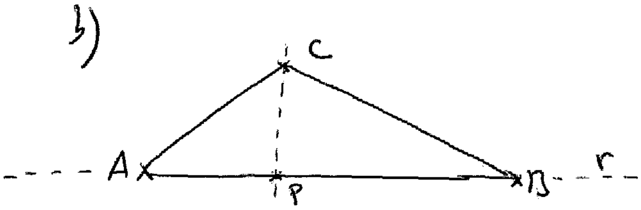
$\begin{array}{l} -3x-6y+21=0 \\ \underline{-12x+6y+66=0} \\ -15x \quad +87=0 \end{array}$; $x = \dfrac{87}{15} = \dfrac{29}{5}$ $\qquad P\left(\dfrac{29}{5}, \dfrac{3}{5}\right)$

Hallo C'.

$\dfrac{C+C'}{2} = P$; $C+C' = 2P$; $C' = 2P - C$; $C' = 2\left(\dfrac{29}{5}, \dfrac{3}{5}\right) - (3,-5)$

$C' = \left(\dfrac{58}{5}, \dfrac{6}{5}\right) - (3,-5)$; $\boxed{C' = \left(\dfrac{43}{5}, \dfrac{31}{5}\right)}$

b)



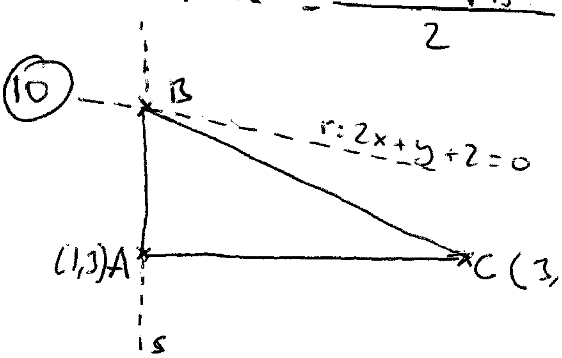
$\text{Área} = \dfrac{\text{base} \times \text{altura}}{2} = \dfrac{|\vec{AB}| \cdot d(C,r)}{2}$

$\vec{AB} = B-A = (6,-3) \rightarrow |\vec{AB}| = \sqrt{36+9} = \sqrt{45}$

$d(C,r) = \dfrac{|-3\cdot 3 - 6\cdot(-5) + 21|}{\sqrt{3^2+6^2}} = \dfrac{|-9+30+21|}{\sqrt{45}} = \dfrac{42}{\sqrt{45}}$

$\text{Área} = \dfrac{\sqrt{45} \cdot \dfrac{42}{\sqrt{45}}}{2} = \dfrac{42}{2} = \boxed{21\ u^2}$

(10)



Hallo la recta s que es la perpendicular al lado AC que pasa por B.

$s: \begin{cases} A(1,3) \\ \vec{v}_s \perp \vec{AC} \Rightarrow \vec{v}_s = (3,2) \end{cases}$ $\quad \dfrac{x-1}{3} = \dfrac{y-3}{2}$

$\vec{AC} = (2,-3)$ $\qquad 2x-2 = 3y-9$

$s: 2x-3y+7=0$

El punto B es el punto de corte de las rectas r y s.

$\left.\begin{array}{l} 2x+y+2=0 \\ 2x-3y+7=0 \end{array}\right\}$ $\begin{array}{l} 2x+y+2=0 \\ \underline{-2x+3y-7=0} \\ \quad 4y-5=0 \end{array}$; $y = \dfrac{5}{4}$ $\qquad \begin{array}{l} 6x+3y+6=0 \\ \underline{2x-3y+7=0} \\ 8x \quad +13=0 \end{array}$; $x = \dfrac{-11}{8}$ $\qquad \boxed{B\left(-\dfrac{13}{8}, \dfrac{5}{4}\right)}$